Multiple dynamic transitions in an anisotropic Heisenberg ferromagnet driven by polarized magnetic field

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A uniaxially (along the Z axis) anisotropic Heisenberg ferromagnet, in the presence of time-dependent (but uniform over space) magnetic field, is studied by Monte Carlo simulation. The time-dependent magnetic field was taken as elliptically polarized where the resultant field vector rotates in the X-Z plane. The system is cooled (in the presence of the elliptically polarized magnetic field) from high temperature. As the temperature decreases, it was found that in the low anisotropy limit the system undergoes three successive dynamical phase transitions. These three dynamic transitions were confirmed by studying the temperature variation of dynamic "specific heat." The temperature variation of dynamic specific heat shows three peaks indicating three dynamic transition points.

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INTRODUCTION

The dynamical behaviors of magnetic model systems, in the presence of a time-dependent magnetic field, show interesting physical phenomena [1]. The nonequilibrium dynamical phase transition [1], particularly in the kinetic Ising model, has drawn much interest of researchers in the field of nonequilibrium statistical physics. The dynamic transition in the kinetic Ising model in the presence of a magnetic field (sinusoidally varying in time) was noticed [2] in the mean-field solution of the dynamical equation for the average magnetization. The time-averaged magnetization over a full cycle (of external magnetic field) becomes nonzero at finite values of temperature and field amplitude. These values of temperature and field amplitudes depend on the frequency of the oscillating field. However, the transition there [2] is not perfectly dynamic in nature since it can exist for such equation of motion even in static (zero-frequency) limit! This reveals that the transition in the zero-frequency limit is an artifact of the mean-field approximation which does not consider the nontrivial fluctuations. The occurrence of the true dynamic transitions for models, incorporating the thermodynamical fluctuations, was later shown in several Monte Carlo studies [1].

After observing the true dynamic transition in the kinetic Ising model in presence of oscillating magnetic field and knowing that it is a nonequilibrium transition, a considerable amount of studies were performed [1] to establish that this transition is thermodynamic phase transition. The divergences of "time scale" [3] and the "dynamic specific heat" [3] and the divergence of length scale [4] are two important observations to establish that the dynamic transition is a thermodynamic phase transition.

Although the dynamic transition in kinetic Ising model is an interesting phenomenon and a simple example to grasp the various features of nonequilibrium phase transitions, it has several limitations. In the Ising model, since the spins can have only two orientations (up/down), some interesting features of dynamic transitions (related to the dynamic transverse ordering) are missing in this model. The classical vector spin model [5] would be the better choice to see such interesting phenomena which are missing in the Ising model. One such example is the "off-axial" dynamic transitions [6] recently observed in the anisotropic Heisenberg ferromagnet. In the off-axial dynamic transition, the dynamical symmetry along the axis of anisotropy can be broken by applying an oscillating field along any perpendicular direction (X direction, say). The dynamic phase transition in an anisotropic XY spin system in an oscillating magnetic field was recently studied [7] by solving the Ginzburg-Landau equation. The dynamic phase transition and the dependence of its behavior on the bilinear exchange anisotropy of a classical Heisenberg spin system (planar thin ferromagnetic film), was recently studied [9] by Monte Carlo simulation.

All these studies on the dynamic phase transition, made so far, are related to a single transition. The dynamic transition occurs at a single value of temperature (for fixed values of field amplitude and frequency). No evidence of multiple dynamic transitions (for bulk only) is reported so far in the literature in an anisotropic Heisenberg ferromagnet driven by a polarized magnetic field. However, it should be mentioned here that a very recent study [8] of dynamical phase transitions in thin Heisenberg ferromagnetic films with bilinear exchange anisotropy has shown multiple phase transitions for the surface and bulk layers of the film at different temperatures. Here, in this paper, the observations of multiple (triple) dynamic transitions, in an anisotropic Heisenberg ferromagnet driven by an elliptically polarized magnetic field, are briefly reported which is observed in the uniaxially anisotropic Heisenberg ferromagnet (three dimensional) in the presence of an elliptically polarized magnetic field studied by Monte Carlo simulations.

The paper is organized as follows: the model is introduced and Monte Carlo simulation techniques are described in the next section, the third section contains numerical results with figures, and the paper ends with a summary and few concluding remarks given in the fourth section.

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MODEL AND MONTE CARLO SIMULATION TECHNIQUE

The Hamiltonian of a classical anisotropic (uniaxial and single-site) Heisenberg model [5], with nearest-neighbor ferromagnetic interaction in the presence of a magnetic field, can be represented as

$$H=-J\sum_{\langle ij\rangle}\mathbf{S}_i\cdot\mathbf{S}_j-D\sum_i (S_i^z)^2-\mathbf{h}\cdot\sum_i \mathbf{S}_i, \qquad (1)$$

where $\mathbf{S}_i[S_{ix},S_{iy},S_{iz}]$ represents a classical spin vector of magnitude unity ($S_{ix}^2+S_{iy}^2+S_{iz}^2=1$) situated at the ith lattice site. The classical spin vector $\mathbf{S}_i$ can be oriented in any (unrestricted) direction in the vector spin space. In the above expression of the Hamiltonian, the first term represents the nearest-neighbor ($\langle ij\rangle$) ferromagnetic ($J>0$) interaction. The factor D in the second term represents the strength of uniaxial (z axis here) anisotropy which is favoring the spin to be aligned along the z axis. Here, it may be noted that for $D=0$, the system is in the isotropic Heisenberg limit and for $D\rightarrow\infty$ the system goes to the Ising limit. The last term stands for the interaction with the externally applied time-dependent magnetic field $[\mathbf{h}(h_x,h_y,h_z)]$. The magnetic-field components are sinusoidally oscillating in time, i.e., $h_\alpha=h_{0\alpha}\cos\omega t$, where $h_{0\alpha}$ is the amplitude and ω is the angular frequency ($\omega=2\pi f$; f is the frequency) of the αth component of the magnetic field. In this present case, the field is taken as elliptically polarized. The polarized field can be represented as

$$\mathbf{h}=\hat{x}h_x+\hat{y}h_y+\hat{z}h_z=\hat{x}h_{0x}\cos(\omega t)+\hat{z}h_{0z}\sin(\omega t). \qquad (2)$$

One can readily check that $h_x=h_{0x}\cos(\omega t)$ and $h_z=h_{0z}\sin(\omega t)$ yield, after the elimination of time,

$$\frac{h_x^2}{h_{0x}^2}+\frac{h_z^2}{h_{0z}^2}=1, \qquad (3)$$

which shows that the magnetic field lies on the X-Z plane and is elliptically polarized (in general, h_{0x} and h_{0z} are not equal). If $h_{0x}=h_{0z}=h_0$ (say), the above equation will take the form $h_x^2+h_z^2=h_0^2$ and the field will be called circularly polarized. The magnetic fields and the strength of anisotropy (D) are measured in the unit of J. The model is defined on a simple cubic lattice of linear size L with periodic boundary conditions applied in all three directions.

The model described above has been studied by Monte Carlo simulation using the following algorithm [10]. To obtain the equilibrium spin configuration at a particular temperature T, the system is slowly cooled down from a random initial spin configuration [11]. At any fixed temperature T (measured in the unit of J/k_B, where k_B is the Boltzmann constant), and for the fixed values of h_{0x}, h_{0z}, ω, and D, a lattice site i has been chosen randomly (random updating scheme). Monte Carlo simulations were performed using the Metropolis algorithm [10] with a random updating scheme. The spin-tilt trial configuration is generated as follows [11,6]: two different random numbers r_1 and r_2 (uniformly distributed between -1 and $+1$), are chosen in such a way that $R^2=r_1^2+r_2^2$ becomes less than or equal to unity. The set of values of r_1 and r_2, for which $R^2>1$, are rejected. Now, $u=\sqrt{1-R^2}$, $S_{ix}=2ur_1$, $S_{iy}=2ur_2$ and $S_{iz}=1-2R^2$. In this way, the distribution of points of tips of spin vectors on the surface of a unit sphere will be uniform. The acceptance of a trial configuration is determined by the Metropolis rate [10]. L^3 numbers of such updates (at random positions) of spin vectors, defines one Monte Carlo step per site (MCSS) and this may be considered as the unit of time in this simulation. The linear frequency ($f=\omega/2\pi$) of the time varying magnetic field is taken 0.02 and kept constant throughout this simulational study. Thus 50 MCSS's are required to obtain one complete cycle of the oscillating field. Consequently, 50 MCSS's is the time period (τ) of the oscillating magnetic field. Any macroscopic quantity, such as any component of magnetization at any instant, is calculated as follows: Starting with an initial random spin configuration (high-temperature phase), the system is allowed to become stabilized (dynamically) up to 4×10^4 MCSS's (i.e., 800 complete cycles of the oscillating field). The average value of various physical quantities are calculated from further 4×10^4 MCSS's (i.e., averaged over another 800 cycles). This is important to achieve stable value and it was checked carefully that the number of MCSS's mentioned above is sufficient to obtain a stable value of the measurable quantities, etc. which can clearly show the dynamic transition points within limited accuracy. But to describe the critical behaviors very precisely (e.g., to estimate critical exponent, etc.) a much longer run is necessary. Here, the total length of simulation for one fixed temperature is 8×10^4. The system is slowly cooled down (T has been reduced by a small interval) to obtain the values of the statistical quantities in the low-temperature ordered phase. The last spin configuration obtained at previous temperature is used as the initial configuration for the new temperature. The CPU time required for 8×10^4 MCSS's is approximately 25 min on an Intel Pentium-III processor.

NUMERICAL RESULTS

The simulational study is done for a simple cubic lattice of linear size $L=20$. The instantaneous magnetization components (per lattice site) $m_x(t)=\Sigma_i(S_x^i/L^3)$, $m_y(t)=\Sigma_i(S_y^i/L^3)$, and $m_z(t)=\Sigma_i(S_z^i/L^3)$ are calculated at each time in the presence of magnetic field. The time averaged (over a full cycle of the oscillating magnetic field) magnetization components (the dynamic order-parameter components) $Q_x=(1/\tau)\oint m_x(t)dt$, $Q_y=(1/\tau)\oint m_y(t)dt$, and $Q_z=(1/\tau)\oint m_z(t)dt$ are calculated by integrating (over the complete cycle of the oscillating field) the instantaneous magnetization components. The total (vector) dynamic order parameter can be expressed as $\mathbf{Q}=iQ_x+jQ_y+kQ_z$. The instantaneous energy $e(t)=-J\Sigma_{\langle ij\rangle}\mathbf{S}_i\cdot\mathbf{S}_j-D\Sigma_i(S_i^z)^2-\mathbf{h}\cdot\Sigma_i\mathbf{S}_i$ is also calculated. The time-averaged instantaneous energy is $E=(1/\tau)\oint e(t)dt$. The rate of change of E with respect to the temperature T is defined as dynamic specific heat $C(=dE/dT)$ [3]. The dynamic specific heat C is calculated from energy E, just by calculating the derivative using the three-point central difference formula, given below.

$$C=\frac{dE}{dT}=\frac{E(T+\delta T)-E(T-\delta T)}{2\,\delta T}. \tag{4}$$

For the elliptically polarized [Eq. (3)] magnetic field, where the resultant field lies in the *X-Z* plane, the amplitudes of fields are taken as $h_{0x}=0.3$ and $h_{0z}=1.0$. The strength of uniaxial anisotropy is taken as $D=0.2$. This value of D is obtained by rigorous searching to have these interesting results and is kept constant throughout the study. However, there must be variations in transition points depending on the values of D. It is observed that for higher values of D the multiple transition phenomenon disappears. The values of field amplitudes and frequency are also obtained by searching.

The temperature variations of the dynamic order-parameter components (Q_x, Q_y, Q_z) are studied and the results are depicted in Fig. 1(a). As the system is cooled down, from a high-temperature disordered $(\mathbf{Q}=\vec{0})$ phase, it was observed that first the system undergoes a transition from dynamically disordered $(\mathbf{Q}=\vec{0})$ to a dynamically *Y*-ordered (only $Q_y\neq 0$) phase. This may be called the first phase (P_1) and the transition temperature is T_{c1}. This phase can be characterized as P_1: $(Q_x=0, Q_y\neq 0, Q_z=0)$. Here, the resultant vector of elliptically polarized magnetic field lies in the *x-z* plane and the dynamic ordering occurs along the *y* direction. So, this is clearly an off-axial transition [6]. In the case of this type of off-axial transition the dynamical symmetry (in any direction; *y* direction here) is broken by the application of the magnetic field in the perpendicular direction (lies in the *x-z* plane here). As the system cools down, it retains this particular dynamically ordered phase (P_1) over a considerable range of temperatures. As the temperature decreases further, a second transition was observed. Here, the system becomes dynamically ordered both in the *X* and *Z* directions at the cost of *Y* ordering. In this new dynamic phase, P_2: $(Q_x\neq 0, Q_y=0, Q_z\neq 0)$. In this phase the dynamical ordering is planar (lies on the *x-z* plane). The ordering occurs in the same plane on which the field vector lies. This transition is axial [6]. This phase may be called the second phase (P_2) and the transition (from first phase to the second phase) temperature is T_{c2}. As the temperature decreases further, the *X* and *Z* ordering increases. At some lower temperature, a third transition was observed, from where the *X* ordering starts to decrease and only *Z* ordering starts to increase quite rapidly. This third phase can be designated as P_3: $(Q_x\neq 0, Q_y=0, Q_z\neq 0)$. Although the characterization of P_2 and P_3, in terms of the values of dynamic order-parameter components, looks similar there exists an important difference between these two phases. In the phase P_2, both Q_x and Q_z increase as the temperature decreases but in the phase P_3, Q_x decreases as the temperature decreases [see Fig. 1(a)]. So these two phases P_2 and P_3 distinctly differ from each other. In this phase the dynamical ordering is also axial (along the *Z* axis or anisotropy axis). The system continues to increase the dynamical *Z* ordering as the temperature decreases further. The low-temperature phase is only dynamically *Z* ordered. That means the systems orders dynamically (only $Q_z\neq 0$) along the *Z* direction (direction of

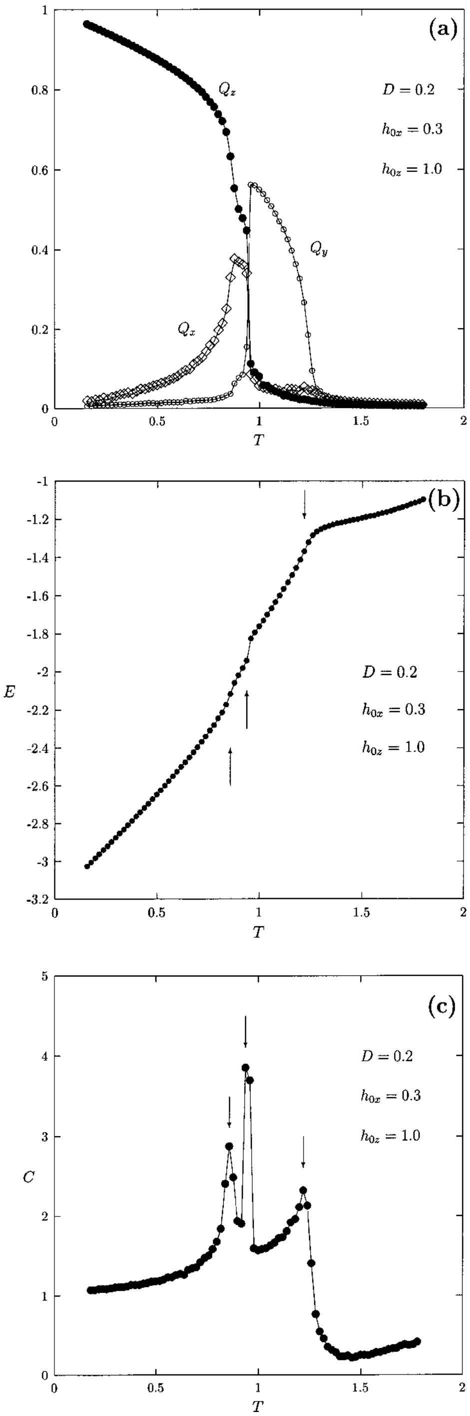

FIG. 1. (a) The temperature variations of the components of dynamic order parameters. Different symbols represent different components. Q_x (◇), Q_y (○), and Q_z (●). This diagram is for $D=0.2$ and for an elliptically polarized field where $h_{0x}=0.3$ and $h_{0z}=1.0$. The size of the error bars of Q_x, Q_y, and Q_z close to the transition points is of the order of 0.02 and that at low temperature (e.g., below $T=0.5$) is around 0.003. (b) The temperature variation of the dynamic energy (E) for $D=0.2$, $h_{0x}=0.3$, and $h_{0z}=1.0$. The vertical arrows represent the transition points. (c) The temperature variation of dynamic specific heat ($C=dE/dT$) for $D=0.2$, $h_{0x}=0.3$, and $h_{0z}=1.0$. Vertical arrows show the peaks and the transition points.

anisotropy) only at very low temperatures. The zero-temperature dynamic phase (for such a polarized field) can be characterized as $Q_x=0$, $Q_y=0$, and $Q_z=1.0$.

To detect the dynamic transitions and to find the transition temperatures the temperature variation of the energy E is plotted in Fig. 1(b). From this figure it is clear that three dynamic transitions occur in this case. The transition points are the inflection points in the E-T curve. The temperature derivative of energy E is the dynamic specific heat C. The temperature variation of C is shown in Fig. 1(c). The three dynamic transitions are very clearly shown by three peaks of the specific heat plotted against the temperature T. From this figure the transition temperatures are calculated (from the peak positions of the C-T curve). The first transition (right peak) occurs around $T_{c1}=1.22$, the second transition (middle peak) occurs at $T_{c2}=0.94$, and the third (left peak) transition occurs around $T_{c3}=0.86$.

This study was further extended for other values of h_{0x} keeping other parameters fixed. It was found that this three-transition scenario disappears for higher values of h_{0x}. For example, for $h_{0x}=0.9$, the second phase P_2 disappears. In this case, the C-T curve shows two peaks. It was also observed that for $h_{0x}=0.2$, $h_{0z}=0.2$ (keeping all other parameter fixed) the system shows a single transition and only dynamically orders along the Z direction.

To detect the dynamic transition points an alternative method may be to study the temperature variation of fluctuation of dynamic order parameter $\chi(Q)[=L^3(\langle Q^2\rangle-\langle Q\rangle^2)]$. However, we do not have a sufficient amount of precise data to study this.

SUMMARY

The uniaxially (Z direction) anisotropic Heisenberg ferromagnet in the presence of a time-dependent (but uniform over space) magnetic field is studied by Monte Carlo simulation using Metropolis dynamics. The dynamic transition in a uniaxially anisotropic Heisenberg ferromagnet has already been studied by Monte Carlo simulation. In that case the time-dependent magnetic field was sinusoidal and the axial and off-axial dynamic transition was reported [6] earlier.

In the present study, the external time-dependent magnetic field was taken as elliptically polarized where the resultant field vector rotates on the X-Z plane. For the lower values of anisotropy and a specific range of the values of field amplitudes the system undergoes multiple dynamic phase transitions. Here, three distinct phases are identified. In this paper, this observation is just briefly reported. This multiple dynamic phase transition in an anisotropic Heisenberg ferromagnet in the presence of an elliptically polarized field is observed here by Monte Carlo simulation. An alternative method to check this phenomenon may be to use the Landau-Lifshitz-Gilbert equation of motion [12] with Langevin dynamics. Another important thing should be mentioned here regarding the possible explanation of multiple dynamic phase transitions (axial and off-axial transitions) observed in the anisotropic Heisenberg model. One possible reason may be the coherent rotation of spins, where the dynamic phase transition in the Ising model can be explained simply by spin reversal and nucleation [13]. But to establish the responsible mechanism behind the multiple dynamic phase transitions, detailed investigations are required.

The variations of the dynamic phase boundaries with frequency and the strength of anisotropy is quite interesting to study. This study also indicates that the system will show a very rich phase diagram with multicritical behavior. The finite-size analysis is also necessary in order to distinguish the crossover effects from the true phase transitions. This requires a huge computational task which will take much time. This work is in progress and the details will be reported later.

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